12.1 Explore Solids

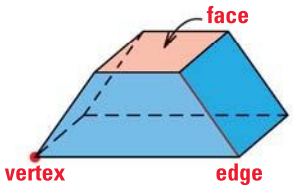
Before You identified polygons.

Now You will identify solids.

Why So you can analyze the frame of a house, as in Example 2.

Key Vocabulary
- **polyhedron**
  face, edge, vertex
- **base**
- **regular polyhedron**
- **convex polyhedron**
- **Platonic solids**
- **cross section**

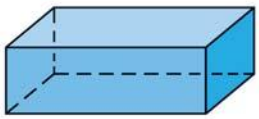
A **polyhedron** is a solid that is bounded by polygons, called **faces**, that enclose a single region of space. An **edge** of a polyhedron is a line segment formed by the intersection of two faces. A **vertex** of a polyhedron is a point where three or more edges meet. The plural of polyhedron is *polyhedra* or *polyhedrons*.



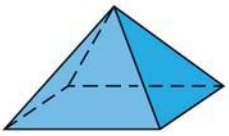
KEY CONCEPT *For Your Notebook*

Types of Solids

Polyhedra

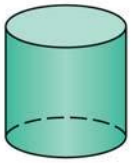


Prism

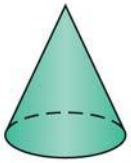


Pyramid

Not Polyhedra



Cylinder

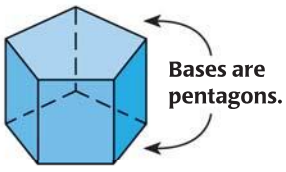


Cone

Sphere

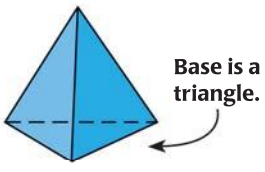
CLASSIFYING SOLIDS Of the five solids above, the prism and the pyramid are polyhedra. To name a prism or a pyramid, use the shape of the *base*.

Pentagonal prism



The two **bases** of a prism are congruent polygons in parallel planes.

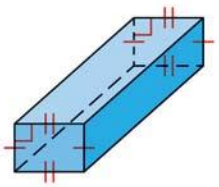
Triangular pyramid

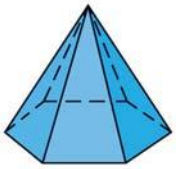


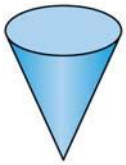
The **base** of a pyramid is a polygon.

EXAMPLE 1 Identify and name polyhedra

Tell whether the solid is a polyhedron. If it is, name the polyhedron and find the number of faces, vertices, and edges.

a. 

b. 

c. 

Solution

a. The solid is formed by polygons, so it is a polyhedron. The two bases are congruent rectangles, so it is a rectangular prism. It has 6 faces, 8 vertices, and 12 edges.

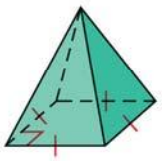
b. The solid is formed by polygons, so it is a polyhedron. The base is a hexagon, so it is a hexagonal pyramid. It has 7 faces, consisting of 1 base, 3 visible triangular faces, and 3 non-visible triangular faces. The polyhedron has 7 faces, 7 vertices, and 12 edges.

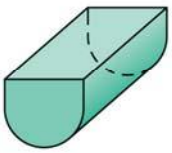
c. The cone has a curved surface, so it is not a polyhedron.

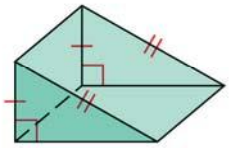
Animated Geometry at classzone.com

GUIDED PRACTICE for Example 1

Tell whether the solid is a polyhedron. If it is, name the polyhedron and find the number of faces, vertices, and edges.

1. 

2. 

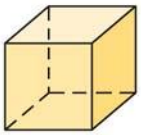
3. 

EULER'S THEOREM Notice in Example 1 that the sum of the number of faces and vertices of the polyhedra is two more than the number of edges. This suggests the following theorem, proved by the Swiss mathematician Leonhard Euler (pronounced "oi'-ler"), who lived from 1707 to 1783.

THEOREM *For Your Notebook*

THEOREM 12.1 Euler's Theorem

The number of faces (F), vertices (V), and edges (E) of a polyhedron are related by the formula $F + V = E + 2$.



$F = 6, V = 8, E = 12$

$6 + 8 = 12 + 2$

EXAMPLE 2 Use Euler's Theorem in a real-world situation

HOUSE CONSTRUCTION **Find the number of edges on the frame of the house.**

Solution

The frame has one face as its foundation, four that make up its walls, and two that make up its roof, for a total of 7 faces.

To find the number of vertices, notice that there are 5 vertices around each pentagonal wall, and there are no other vertices. So, the frame of the house has 10 vertices.

Use Euler's Theorem to find the number of edges.

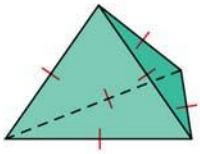
$F + V = E + 2$ **Euler's Theorem**

$7 + 10 = E + 2$ **Substitute known values.**

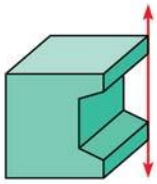
$15 = E$ **Solve for *E*.**

▸ The frame of the house has 15 edges.

REGULAR POLYHEDRA A polyhedron is **regular** if all of its faces are congruent regular polygons. A polyhedron is **convex** if any two points on its surface can be connected by a segment that lies entirely inside or on the polyhedron. If this segment goes outside the polyhedron, then the polyhedron is nonconvex, or *concave*.



regular, convex

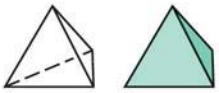


nonregular, concave

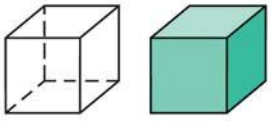
There are five regular polyhedra, called **Platonic solids** after the Greek philosopher Plato (c. 427 B.C.–347 B.C.). The five Platonic solids are shown.

READ VOCABULARY

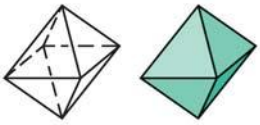
Notice that the names of four of the Platonic solids end in "hedron." *Hedron* is Greek for "side" or "face." Sometimes a cube is called a regular *hexahedron*.



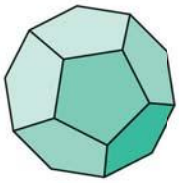
Regular tetrahedron
4 faces



Cube
6 faces



Regular octahedron
8 faces



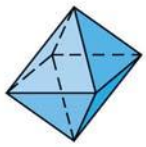
Regular dodecahedron
12 faces

Regular icosahedron
20 faces

There are only five regular polyhedra because the sum of the measures of the angles that meet at a vertex of a convex polyhedron must be less than 360°. This means that the only possible combinations of regular polygons at a vertex that will form a polyhedron are 3, 4, or 5 triangles, 3 squares, and 3 pentagons.

EXAMPLE 3 Use Euler's Theorem with Platonic solids

Find the number of faces, vertices, and edges of the regular octahedron. Check your answer using Euler's Theorem.



ANOTHER WAY

An octahedron has 8 faces, each of which has 3 vertices and 3 edges. Each vertex is shared by 4 faces; each edge is shared by 2 faces. They should only be counted once.

$V = \frac{8 \cdot 3}{4} = 6$

$E = \frac{8 \cdot 3}{2} = 12$

Solution

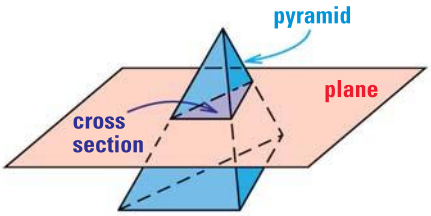
By counting on the diagram, the octahedron has 8 faces, 6 vertices, and 12 edges. Use Euler's Theorem to check.

$F + V = E + 2$ Euler's Theorem

$8 + 6 = 12 + 2$ Substitute.

$14 = 14$ ✓ This is a true statement. So, the solution checks.

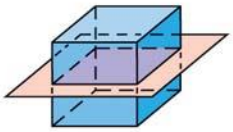
CROSS SECTIONS Imagine a plane slicing through a solid. The intersection of the plane and the solid is called a **cross section**. For example, the diagram shows that an intersection of a plane and a triangular pyramid is a triangle.



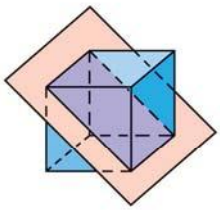
EXAMPLE 4 Describe cross sections

Describe the shape formed by the intersection of the plane and the cube.

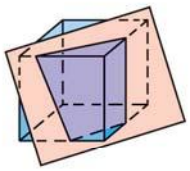
a.



b.



c.



Solution

a. The cross section is a square.

b. The cross section is a rectangle.

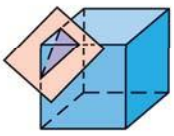
c. The cross section is a trapezoid.

✓ GUIDED PRACTICE for Examples 2, 3, and 4

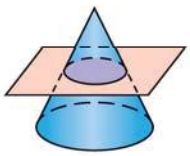
4. Find the number of faces, vertices, and edges of the regular dodecahedron on page 796. Check your answer using Euler's Theorem.

Describe **the shape formed by the intersection of the plane and the solid.**

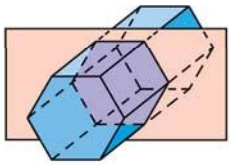
5.



6.



7.



12.1 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 11, 25, and 35

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 21, 28, 30, 31, 39, and 41

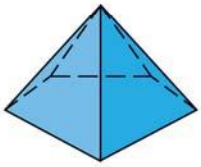
SKILL PRACTICE

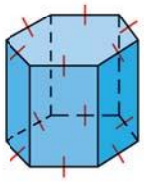
1. **VOCABULARY** Name the five Platonic solids and give the number of faces for each.

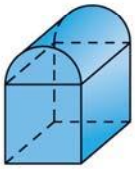
2. ★ **WRITING** State Euler's Theorem in words.

EXAMPLE 1 on p. 795 for Exs. 3–10

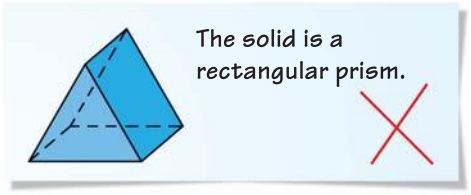
IDENTIFYING POLYHEDRA **Determine whether the solid is a polyhedron. If it is, name the polyhedron. *Explain* your reasoning.**

3. 

4. 

5. 

6. **ERROR ANALYSIS** *Describe* and correct the error in identifying the solid.



SKETCHING POLYHEDRA **Sketch the polyhedron.**

7. Rectangular prism
8. Triangular prism
9. Square pyramid
10. Pentagonal pyramid

EXAMPLES 2 and 3 on pp. 796–797 for Exs. 11–24

APPLYING EULER'S THEOREM **Use Euler's Theorem to find the value of n.**

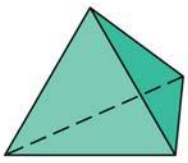
11. Faces: n
    Vertices: 12
    Edges: 18

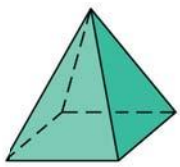
12. Faces: 5
    Vertices: n
    Edges: 8

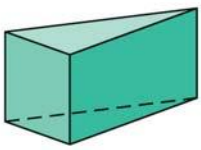
13. Faces: 10
    Vertices: 16
    Edges: n

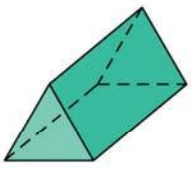
14. Faces: n
    Vertices: 12
    Edges: 30

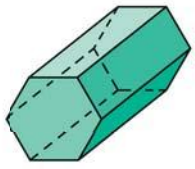
APPLYING EULER'S THEOREM **Find the number of faces, vertices, and edges of the polyhedron. Check your answer using Euler's Theorem.**

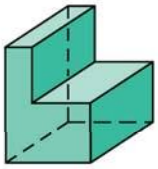
15. 

16. 

17. 

18. 

19. 

20. 

21. ★ **WRITING** *Explain* why a cube is also called a regular hexahedron.

PUZZLES **Determine whether the solid puzzle is *convex* or *concave*.**

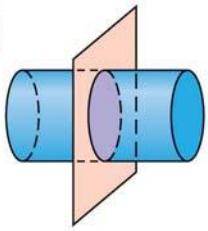
22. 

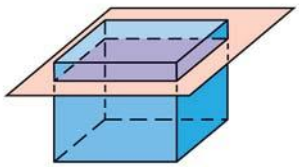
23. 

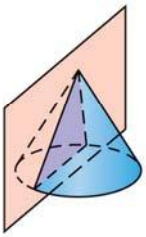
24. 

EXAMPLE 4
on p. 797
for Exs. 25–28

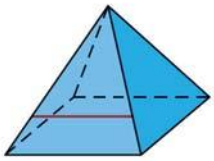
CROSS SECTIONS **Draw and *describe* the cross section formed by the intersection of the plane and the solid.**

25. 

26. 

27. 

28. ★ **MULTIPLE CHOICE** What is the shape of the cross section formed by the plane parallel to the base that intersects the red line drawn on the square pyramid?



(A) Square (B) Triangle

(C) Kite (D) Trapezoid

29. **ERROR ANALYSIS** *Describe* and correct the error in determining that a tetrahedron has 4 faces, 4 edges, and 6 vertices.

30. ★ **MULTIPLE CHOICE** Which two solids have the same number of faces?

(A) A triangular prism and a rectangular prism

(B) A triangular pyramid and a rectangular prism

(C) A triangular prism and a square pyramid

(D) A triangular pyramid and a square pyramid

31. ★ **MULTIPLE CHOICE** How many faces, vertices, and edges does an octagonal prism have?

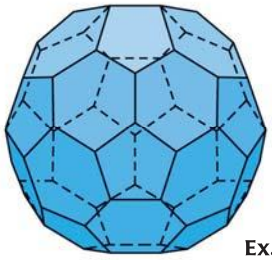
(A) 8 faces, 6 vertices, and 12 edges

(B) 8 faces, 12 vertices, and 18 edges

(C) 10 faces, 12 vertices, and 20 edges

(D) 10 faces, 16 vertices, and 24 edges

32. **EULER'S THEOREM** The solid shown has 32 faces and 90 edges. How many vertices does the solid have? *Explain* your reasoning.



Ex. 32

33. **CHALLENGE** *Describe* how a plane can intersect a cube to form a hexagonal cross section.

PROBLEM SOLVING

EXAMPLE 2 on p. 796 for Exs. 34–35

34. MUSIC The speaker shown at the right has 7 faces. Two faces are pentagons and 5 faces are rectangles.

a. Find the number of vertices.

b. Use Euler's Theorem to determine how many edges the speaker has.

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35. CRAFT BOXES The box shown at the right is a hexagonal prism. It has 8 faces. Two faces are hexagons and 6 faces are squares. Count the edges and vertices. Use Euler's Theorem to check your answer.

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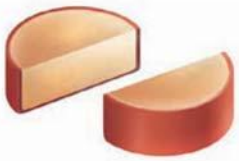
FOOD ***Describe*** **the shape that is formed by the cut made in the food shown.**

36. Watermelon

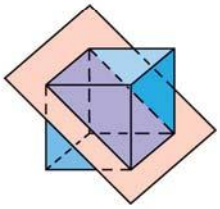
37. Bread

38. Cheese



39. ★ SHORT RESPONSE Name a polyhedron that has 4 vertices and 6 edges. Can you draw a polyhedron that has 4 vertices, 6 edges, and a different number of faces? *Explain* your reasoning.

40. MULTI-STEP PROBLEM The figure at the right shows a plane intersecting a cube through four of its vertices. An edge length of the cube is 6 inches.

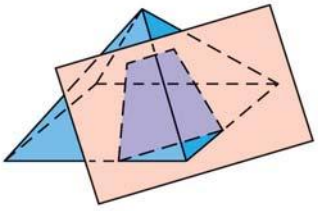


a. *Describe* the shape formed by the cross section.

b. What is the perimeter of the cross section?

c. What is the area of the cross section?

41. ★ EXTENDED RESPONSE Use the diagram of the square pyramid intersected by a plane.



a. *Describe* the shape of the cross section shown.

b. Can a plane intersect the pyramid at a point? If so, sketch the intersection.

c. *Describe* the shape of the cross section when the pyramid is sliced by a plane parallel to its base.

d. Is it possible to have a pentagon as a cross section of this pyramid? If so, draw the cross section.

42. PLATONIC SOLIDS Make a table of the number of faces, vertices, and edges for the five Platonic solids. Use Euler's Theorem to check each answer.

◯ = WORKED-OUT SOLUTIONS on p. WS1

★ = STANDARDIZED TEST PRACTICE

REASONING **Is it possible for a cross section of a cube to have the given shape? If yes, *describe* or sketch how the plane intersects the cube.**

43. Circle **44.** Pentagon **45.** Rhombus

46. Isosceles triangle **47.** Regular hexagon **48.** Scalene triangle

49. **CUBE** *Explain* how the numbers of faces, vertices, and edges of a cube change when you cut off each feature.

a. A corner **b.** An edge **c.** A face **d.** 3 corners

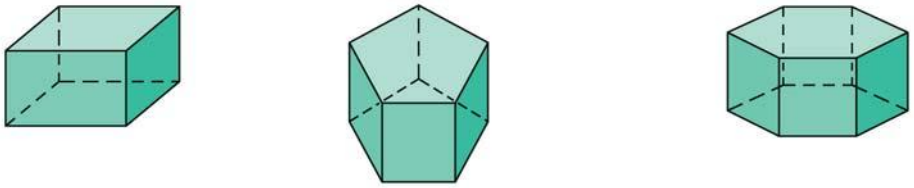
50. **TETRAHEDRON** *Explain* how the numbers of faces, vertices, and edges of a regular tetrahedron change when you cut off each feature.

a. A corner **b.** An edge **c.** A face **d.** 2 edges

51. **CHALLENGE** The *angle defect* D at a vertex of a polyhedron is defined as follows:

$D = 360° -$ (sum of all angle measures at the vertex)

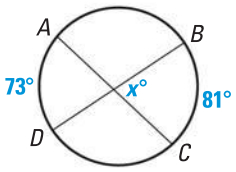
Verify that for the figures with regular bases below, $DV = 720$ where V is the number of vertices.



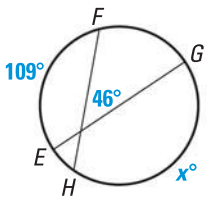
MIXED REVIEW

Find the value of x. *(p. 680)*

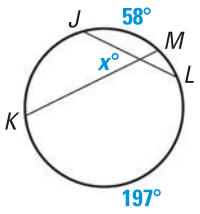
52.



53.



54.



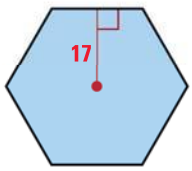
PREVIEW
Prepare for Lesson 12.2 in Exs. 55–60.

Use the given radius r or diameter d to find the circumference and area of the circle. Round your answers to two decimal places. *(p. 755)*

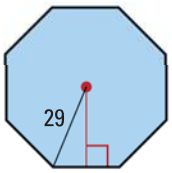
55. $r = 11$ cm **56.** $d = 28$ in. **57.** $d = 15$ ft

Find the perimeter and area of the regular polygon. Round your answers to two decimal places. *(p. 762)*

58.



59.



60.